TABLE I
THEORETICAL CONSTANTS FOR THE *E*-PLANE SYMMETRICAL T JUNCTION

| | hybrid elements | | | | (HssHee-Hse²)/Hss |
|---|---|---|---|---|---|
| b/λg | Hss | 2 Hee | √2 Hse | Hoo | ( = -tan κ lee ) |
| 0.01 | 218. | 218 | -218. | 0.00465 | 0 |
| 0.05 | 42.3 | 41.9 | -42.1 | 0.0231 | 0.0 |
| 0.10 | 19.3 | 18.6 | -18.9 | 0.0452 | 0.0 |
| 0.15 | 11.2 | 10 2 | -10.7 | 0.0655 | 0 0 |
| 0.20 | 7.06 | 6.02 | -6.51 | 0 0825 | 0.01 |
| 0.25 | 4.63 | 3 61 | -4.08 | 0.0960 | 0 01 |
| 0.30 | 3.08 | 2.17 | -2.57 | 0.104 | 0.01 |
| 0.35 | 2.04 | 1 28 | -1 59 | 0.105 | 0 02 |
| 0.40 | 1.30 | 0.729 | -0.934 | 0.0985 | 0 029 |
| 0 425 | 0.994 | 0.535 | -0.683 | 0 0915 | 0 033 |
| 0 45 | 0.713 | 0.378 | -0.465 | 0.0795 | 0 037 |

with

$$H_{MN} \equiv \kappa \int_{-b/2}^{b/2} f_M(z) \epsilon_N(z)\, dz \qquad (M, N = s, e, o). \quad (16)$$

It is easily proved with the aid of (11) that the hybrid matrix is symmetric: $H_{MN} = H_{NM}$. The consideration of the parity of $f_M(z)$ and $\epsilon_N(z)$ immediately produces $H_{so} = H_{eo} = 0$. So the hybrid matrix has four independent elements. Note that the diagonal elements are equal to the inverse stationary values: $H_{MM} = 1/B_M$. On the basis of the circuit equations (15a) and (15b) we can represent the $E$-plane tee by a two-port circuit and a one-port circuit. Fig. 2 is an example.

The numerical analysis of (11) and (16) produces Table I. Here $\lambda_g \equiv 2\pi/\kappa$. The last column represents the determinant of the matrix in (15a) divided by $H_{ss}$, which is equal to $-\tan \kappa l_{ee}$ in Fig. 2. Each value of this table is accurate to within an error of $\pm 1$ in its last place over most of $0 < b/\lambda_g < 0.5$. Using this table, we can calculate the parameters of Fig. 2, ACC's parameters, Sharp's admittance parameters, and so on. We have ascertained that our calculation of ACC's characteristic points coincides with ACC's data [1, table 2] to within 0.4 percent over most of $0 < b/\lambda_g < 0.5$. This agreement is seen to be excellent, and it can be concluded that our numerical analysis is verified by the independent result. The table suggests

$$H_{ee}/H_{ss} \to 1/2 \qquad H_{se}/H_{ss} \to -1/\sqrt{2} \qquad \text{for } b/\lambda_g \to 0. \quad (17)$$

Although the number of figures in the last column is not sufficient for small values of $b/\lambda_g$, eq. (17) leads to $(H_{ss}H_{ee} - H_{se}^2) \to 0$, so $\tan \kappa l_{ee} \to 0$. Noting this asymptotic behavior, we can conclude from the last column that $|\tan \kappa l_{ee}| \ll 1$ over most of $0 < b/\lambda_g < 0.5$. Thus we can eliminate the element $l_{ee}$ as a good approximation, and write Lewin's equivalent circuit. Leaving out $\tan \kappa l_{ee}$, we obtain very simple expressions for the other elements of Fig. 2:

$$b \cong 1/H_{ss} = B_s \qquad \tan \kappa l_{oo} = 1/H_{oo} = B_o$$

$$n^2 \cong (H_{se}/H_{ss})^2 \cong H_{ee}/H_{ss} = B_s/B_e. \quad (18)$$

It is easy to ascertain that these approximate expressions agree with Lewin's expressions ((7a)–(7c)).

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# Effect of a Cochannel Signal on the Electrical Tuning Characteristics of a Gunn Oscillator


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*Abstract* —New experimental observations on the pull-in as well as hold-in characteristics of an injection-locked microwave (Gunn) oscillator in the presence of a cochannel signal have been reported. The analytical treatment presented herein confirms the observations.


## I. INTRODUCTION

The effect of a cochannel signal on the performance of a microwave receiver has been studied in the past by a number of workers [1]–[5]. All these works indicate that the interfering signal reduces the tracking capability of an injection synchronized oscillator (ISO). However, experimental results in some cases show considerable departure from the existing concept. In [2] the effect of a low-strength interfering tone on the tracking behavior of an injection-locked oscillator has been considered, and naturally the amplitude perturbation of the oscillator has been neglected. This study found that the presence of interference induces asymmetry in the one-sided locking range of the oscillator. However, the two-sided locking range remains almost constant. In [3]–[5], however, the effect of large interference has been considered. It has been shown that the locking range decreases with an increase of the interference-to-carrier ratio. But it appears that detailed experimental observations on the performance characteristics of a microwave ISO, such as shift of the center frequency of the oscillator in the unlocked condition and the effect of amplitude perturbation on the locking behavior of an ISO in the presence of a cochannel signal having different detunings from the oscillator frequency, have not been reported. The purpose of the present paper is therefore to critically examine both theoretically and experimentally these characteristics, leading to some new findings.

## II. EXPERIMENT

To study the effect of a cochannel signal on the performance characteristics of an ISO, an experimental arrangement as shown in Fig. 1(a) is set up. Here, a signal generator and a Gunn


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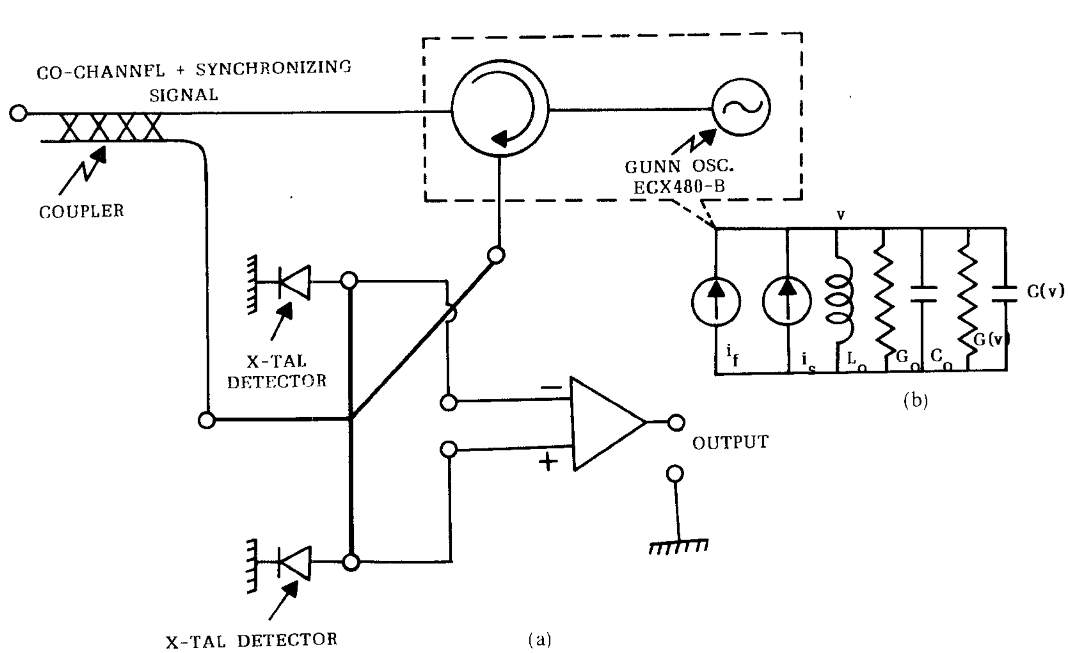


Fig 1. (a) Experimental arrangement of injection synchronized Gunn oscillator exposed to a cochannel signal and (b) its equivalent circuit

oscillator (diode MA 49158) are used as, respectively, the input synchronizing signal source and the cochannel signal source. The input signal accompanied by the cochannel signal is fed to the local oscillator (diode MA 49106) through a circulator. The output of the circulator feeds the E arm of a magic tee. An appreciable fraction of the combined incoming signal is fed to the H arm of the magic tee. The outputs of the collinear arms feed two tunable detectors having square-law characteristics. Finally, the outputs from the detectors are differenced to realize the net output, which can be observed in an oscilloscope. In this experimental setup, the following observations are made.

i) The shift of the center frequency of the oscillator with the strength of the synchronizing signal is measured (Fig. 2(a)) when the frequencies of both the synchronizing signal and the cochannel signal are kept outside the lock band of the oscillator.
ii) The variation of the lock band of the oscillator with the strength of the cochannel signal is measured (Fig. 2(b)).

The experimental characteristics thus observed are explained in the following sections.

## III. Derivation of the System Equations

In order to derive the system equations, we consider the analytical equivalent circuit of the oscillator [6], which is shown in Fig. 1(b). Here, $L_0$, $C_0$, and $G_0$ are the inductance, capacitance, and conductance of the cavity and $C(v)$ and $G(v)$ are the voltage-dependent capacitance and conductance of the device, respectively; and $i_s$ and $i_f$ are, respectively, the synchronizing signal current and the cochannel signal current. Thus, the total instantaneous circuit current is given by

$$i=\frac{1}{L_0}\int v\,dt+G_0v+\frac{d}{dt}[C_0v]+G(v)\cdot v+\frac{d}{dt}[C(v)\cdot v] \quad (1)$$

where $v$ is the output voltage of the oscillator and $i=i_s+i_f$. Now, putting $C(v)/C_0=\alpha_0+\alpha_1v+\alpha_2v^2$, $G(v)/G_0=-\beta_0-\beta_1v+\beta_2v^2$, $\omega_0^2=1/L_0C_0$, $Q=\omega_0C_0/G_0$, $v_0=i/G_0$ and using (1), the system equation can be rewritten as

$$\frac{dv_0}{dt}=\frac{Q}{\omega_0}\frac{d^2}{dt^2}\left[(1+\alpha_0)v+\alpha_1v^2+\alpha_2v^3\right] +\frac{d}{dt}\left[(1-\beta_0)v-\beta_1v^2+\beta_2v^3\right]+\omega_0Qv. \quad (2)$$

Here, the $\alpha$'s and $\beta$'s are the constants for the nonlinear device impedance. Assuming the synchronizing signal to be $E_s\cos(\omega_1t+\theta)$ and the cochannel signal to be $xE_s\cos\omega_tt$, $\theta$ being the input angle modulation, the effective input signal is written as

$$v_0=E_s[\cos(\omega_1t+\theta)+x\cos\omega_tt]. \quad (3)$$

Taking the solution of (2) as

$$v=A(t)\cos(\omega_0t+\Psi(t)) \quad (4)$$

where $A(t)$ and $\Psi(t)$ are slowly varying functions of time, and applying the method of quasi-static and quasi-linearization technique [7] it is not difficult to arrive at the instantaneous amplitude and phase equation of the oscillator:

$$\frac{da}{dt}=\frac{\omega_0}{2Q}(\beta_0-1)(1-a^2)a+K[\cos\varphi+x\cos(\Delta\omega t+\varphi)] \quad (5)$$

and

$$\frac{d\varphi}{dt}=\Omega-\frac{K}{a}[\sin\varphi+x\sin(\Delta\omega t+\varphi)]+\frac{d\theta}{dt} \quad (6)$$

where

$$\varphi=(\omega_1-\omega_0)t+\theta-\Psi$$
$$\Delta w=\omega_t-\omega_1$$
$$\Omega\simeq\omega_1-\omega_0+\frac{\omega_0\alpha_0}{2}+\frac{3}{8}\omega_0\alpha_2A_0^2$$
$$K=\frac{E_s\omega_1}{2QA_0}\simeq\frac{E_s\omega_0}{2QA_0}$$
$$a=A/A_0$$
$$A_0^2=\frac{4}{3}\frac{(\beta_0-1)}{\beta_2}.$$

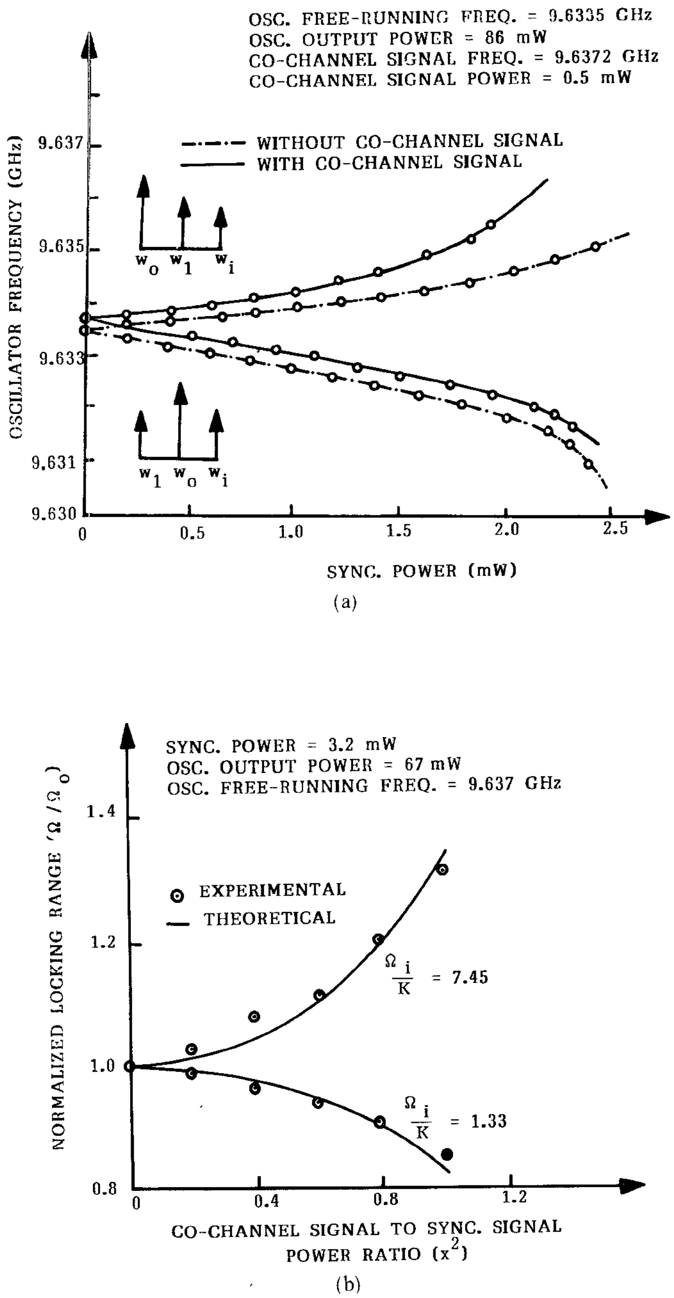


Fig. 2. (a) The pull-in characteristics of a Gunn oscillator in the unlocked condition with the strength of the synchronizing signal ($\Omega = 3$ MHz, $K = 2.5$ MHz). (b) Variation of the locking bandwidth with the strength of the interfering signal ($\Omega_0/K$ = interference-free lock range).

Note that $\omega_0$ indicates the free-running frequency of the oscillator, $\Omega$ is the open-loop frequency error between the oscillator and the synchronizing signal, $\Delta\omega$ denotes the interference detuning from the synchronizing signal, and $K$ denotes the one-sided locking range.

## IV. Unlocked Behavior

It is important to study the behavior of an unlocked oscillator, because it indicates certain important aspects of frequency entrainment. In the following we consider both the frequency of the synchronizing signal and the cochannel signal to be just outside the lock band of the oscillator.

The shift of the center frequency of an oscillator due to pull-in by a synchronizing signal, which is situated just outside the lock band of the oscillator, is well known. But the combined effect of the synchronizing signal and the cochannel signal on the shift of the center frequency of an oscillator when both of them are located outside the lock band of the ISO has not been discussed in the literature.

As the oscillator is operating in the unlocked condition (i.e., $\Omega > K$), the instantaneous phase of the oscillator will be modulated both by the synchronizing signal and by the cochannel signal. As such, the output may be approximated as

$$v(t) = A\cos[(\omega_0 + \Delta)t - M_1\cos\Omega_1 t - M_2\cos\Omega_2 t] \quad (7)$$

where

$$\Omega_1 = \omega_1 - \omega_0 - \Delta = \Omega - \Delta$$

= modulation frequency due to the synchronizing signal

and

$$\Omega_2 = \omega_i - \omega_0 - \Delta = \Omega_i - \Delta$$

= modulation frequency due to the cochannel signal.

Here $\Delta$ is the shift of the center frequency of the oscillator in the unlocked condition, and $M_1$ and $M_2$ are the phase modulation indices. The phase difference ($\varphi$) between the output and the input of the oscillator becomes

$$\varphi = \Omega_1 t + M_1\cos\Omega_1 t + M_2\cos\Omega_2 t. \quad (8)$$

Using (6) and (8), it can be shown that

$$\Delta = K[J_0(M_2)J_1(M_1) + xJ_0(M_1)J_1(M_2)] \quad (9)$$

$$\Omega_1 M_1 = K[J_0(M_1)J_0(M_2)] \quad (10)$$

and

$$\Omega_2 M_2 = xK[J_0(M_1)J_0(M_2)] \quad (11)$$

where $J_n(M)$ is a Bessel function of order $n$ and argument $M$. Since $M_1$ and $M_2$ are small, one can obtain from (9), (10), and (11) an expression for the shift of the center frequency of the oscillator due to the combined effect of the synchronizing signal and the cochannel signal. It can be shown to be given by

$$\Delta = \frac{K^2}{\Omega_1\Omega_2}[\Omega_2 + \Omega_1 x^2]. \quad (12)$$

Theoretical results based on (12) are superimposed on the experimental results depicted in Fig. 2(a).

## V. Locked Behavior

In this section, the frequency entrainment property of the oscillator in the presence of a cochannel signal is studied in detail. We start with the locked condition when the cochannel signal lies outside the lock band and the synchronizing signal is free from angle modulation ($\theta = 0$).

In the free-running mode, the normalized amplitude '$a$' of the oscillator can be taken to be unity, but in the forced condition '$a$' changes to $(1+\Delta a)$, where $\Delta a \ll 1$. This affects the phase equation. Thus both the phase and the amplitude equations have to be considered to evaluate the locking band of the oscillator, and an analytical solution in this regard is not possible. To simplify the situation, we derive in the following an equivalent phase equation that takes into account the effect of amplitude perturbation.

Replacing '$a$' by $(1+\Delta a)$, (5) becomes

$$\frac{d}{dt}\Delta a = -\frac{\omega_0}{Q}(\beta_0 - 1)\Delta a + K[\cos\varphi + x\cos(\Delta\omega t + \varphi)]. \quad (13)$$

In the periodic mode of operation, it may be assumed that

$$\frac{d}{dt}\Delta a \ll \frac{\omega_0}{Q}(\beta_0 - 1)\Delta a.$$

Therefore, one can write from (13)

$$\Delta a = Z[\cos\varphi + x\cos(\Delta\omega t + \varphi)] \tag{14}$$

where

$$Z = \frac{E_s}{2A_0(\beta_0 - 1)}.$$

Using (14) and substituting $(1+\Delta a)$ for '$a$' one can approximate (6) as

$$\frac{d\varphi}{dt} = \Omega + \frac{rZ^2}{2}(1+x^2) - Km_1[\sin\varphi + x\sin(\Delta\omega t + \varphi)] \tag{15}$$

where

$$m_1 = 1 + \frac{Z^2}{2}(1+x^2).$$

It is assumed that the steady-state solution of (15) is

$$\varphi = \varphi_0 + m\sin(\Delta\omega t + \alpha) \tag{16}$$

where $\varphi_0$ is the static (i.e., dc) phase error, $m$ is the phase modulation index, and $\alpha$ is a constant. Considering $x<1$, one can arrive at the following relations:

$$\frac{\Omega}{K} + \frac{rZ^2}{2K}(1+x^2) = m_1\left[J_0(m)\sin\varphi_0 + \frac{\frac{m\Delta\omega}{K}J_1(m)}{m_1 J_0(m)}\right] \tag{17}$$

and

$$\left(\frac{m\Delta\omega}{K}\right)^2 + (2J_1(m)m_1\cos\varphi_0)^2 = (xm_1J_0(m))^2. \tag{18}$$

The value of $\varphi_0$ for the synchronization boundary can be taken as $\pm(\pi/2 - m)$. Therefore, relations (17) and (18) are transformed to

$$\frac{\Omega}{K} + \frac{rZ^2}{2K}(1+x^2) = \pm m_1 + \frac{m^2}{2}\left[\frac{\Delta\omega}{K} \mp m_1\right] \tag{19}$$

and

$$m^4m_1^4\left(1 - \frac{x^2}{16}\right) + m^2\left[\left(\frac{\Delta\omega}{K}\right)^2 + \frac{m_1^2x^2}{2}\right] - m_1^2x^2 = 0. \tag{20}$$

Putting

$$\Delta\omega = \Omega_i - \Omega \qquad Z_0 = \frac{\Omega_i - \Omega}{K}$$

and using the value of $m^2$ from (20), one can write (19) as

$$\frac{\Omega_i}{K} \mp m_1 + \frac{rZ^2}{2K}(1+x^2) = Z_0 + \frac{1}{4m_1^2}[Z_0 \mp m_1]\left[-\left(Z_0^2 + \frac{m_1^2x^2}{2}\right) + \left(\left(Z_0^2 + \frac{m_1^2x^2}{2}\right) + 4m_1^4x^2\right)^{1/2}\right]. \tag{21}$$

This relation is used to calculate the locking bandwidth of the ISO in the presence of the cochannel signal. Fig. 2(b) depicts the variation of the locking bandwidth of a Gunn oscillator with the strength of the cochannel signal for a given synchronizing power and for two different types of interference detuning. It is observed that when $\Omega_i/K \gg 1$, one obtains an increase in the locking range with the strength of the interference, whereas when $\Omega_i/K$ is just greater than unity, one obtains a reduction in the locking range of the oscillator with the strength of the cochannel signal; i.e., simply by changing the detuning of the cochannel signal, one can change the performance characteristics relating to the tracking capability of the ISO.

## VI. Conclusion

(i) There is an additional shift in the oscillator frequency when the incoming signal is contaminated with the cochannel signal. This effect will be greater when both the synchronizing signal and the cochannel signal are on the same side of the oscillator frequency.

(ii) Simply by varying the frequency of the cochannel signal, one can increase or decrease the tracking capability of the ISO with the strength of the cochannel signal.

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# Accurate Characterization of Microstrip Resonator Open End with New Current Expression in Spectral-Domain Approach

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*Abstract* —**This paper describes a resonant frequency characterization of microstrip resonators which is suitable for very accurate computer-aided design. First, the convergence behavior of the numerical computation based on the spectral-domain approach is discussed to secure the convergence accuracy. To reduce computation time considerably, which at present may amount to several hours, a new current expression in the spectral-domain approach is proposed. In the process described, the computational results under a good convergence have excellent agreement with those measured after estimation of a suitable dielectric constant.**

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